Supplemental information:
Simultaneous creation of multiple vortex-antivortex pairs in momentum space in photonic lattices

1 Determination of reciprocal lattice

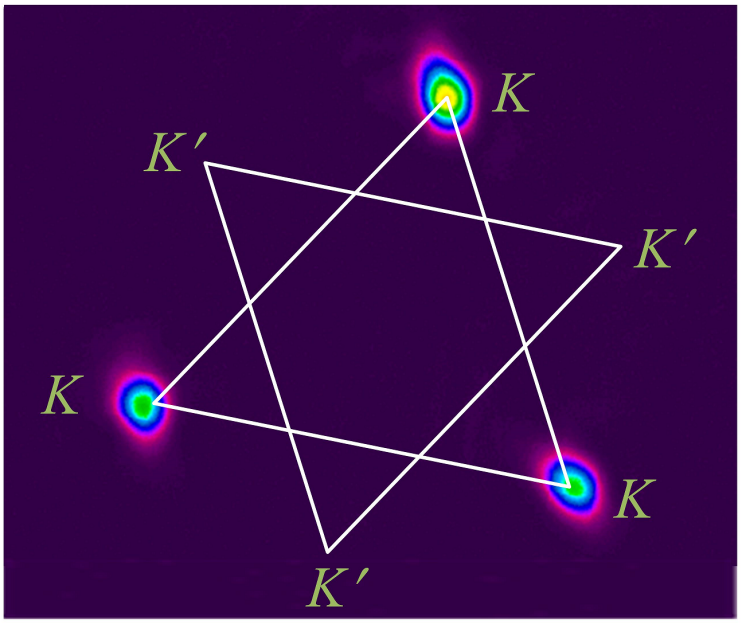

Figure S1: Coupling field $\boldsymbol{E_2}$ intensity profile in momentum space.

2 Simulation details

2.1 Paraxial approximation and Schrödinger equation

The field profile in the cell can be written in general form as

$$\vec{E}(x,y,z,t) = \vec{E_0} a(x,y,z) e^{i(k_0 n z - \omega t)}, \tag{1}$$

where ω is the frequency of the laser beam, n is the refraction coefficient, k_0 is the wave vector of light in the vacuum, $\vec{E_0}$ is the amplitude vector, and a defines the slowly varying profile of the electric field.

In the paraxial approximation $\partial^2 a/\partial z^2 \ll k_0 \partial a/\partial z$ the electric field envelope a obeys the parabolic partial differential equation

$$i\partial_z a = \left[-\frac{\Delta}{2k_0 n_0} - k_0^2 \left(n^2 - n_0^2 \right) \right] a, \tag{2}$$

where $\Delta = \partial_x^2 + \partial_y^2$, $n = n(x,y)$ is the laterally varying part of the refractive index due to the optical modulation with the coupling field, n_0=1 is the background refraction index.

Equation (2) is equivalent to the *time-dependent Schrödinger equation* with z coordinate playing the role of time, with the mass given by $m = \hbar k_0 n_0/c$, and the potential energy $U(x, y)$ determined by the spatial variation of the refraction index: $U(x, y) = -\hbar c k_0^2(n^2 - n_0^2)$, where c is the speed of light. The initial condition for the envelope field a is governed by the Gaussian profile of the probe beam entering the vapor cell (at $z = 0$). The time-dependent Eq. (2) with the honeycomb lattice potential was solved numerically in the general case of spatially varying effective potential lattice $U(x, y)$ and in the tight-binding approximation.

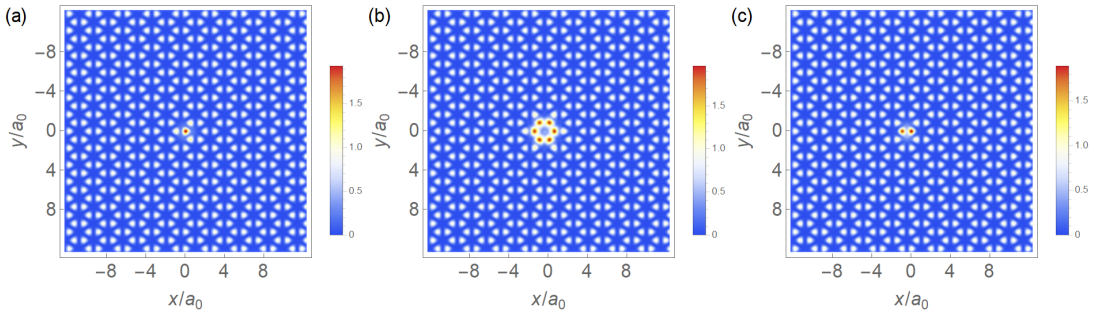

Figure S2: Initial states for the Schrödinger equation simulation for various symmetries of wave function C_{3v} (a), C_{6v} (b), C_{2v} (c). The magnitudes of honeycomb-type potential profile and initial wave function absolute value are normalized to unity and summed.

2.2 Tight binding model calculations

To test the robustness of the results and their sensitivity to the particular shape of the site potential, the *time-dependent tight-binding-type model* (TBM) calculations on the honeycomb lattice were performed. The corresponding equation writes as

$$i\hbar\partial_t\psi_i = t\sum_{\{j\}}\psi_j, \tag{3}$$

where ψ_i is the wave function at site i, t is the hopping parameter and j-summation is conducted over the 3 nearest neighbors of the site i. The eigenstates $\psi^{(\nu)}$ for the finite-size graphene crystal were obtained by numerical diagonalization of the tight-binding Hamiltonian. The initial wave function $\psi(t = 0)$ was projected onto these eigenstates: $\psi(t = 0) = \sum c_\nu\psi^{(\nu)}$. Due to the linearity of the system, the wave function time dependence writes as

$$\psi_i(t) = \sum_\nu c_\nu\psi^{(\nu)} \cdot \exp(-i\omega_\nu t). \tag{4}$$

The results of the TBM were then converted into real-space field using the assumption of uniform WF distribution at each site for direct comparison of the two models. Spline interpolation of the WF using the TBM results was also employed, both approaches showed qualitatively similar results, confirming the validity of the model.

3 Existence of vortices for C_{3v} symmetry tetramer

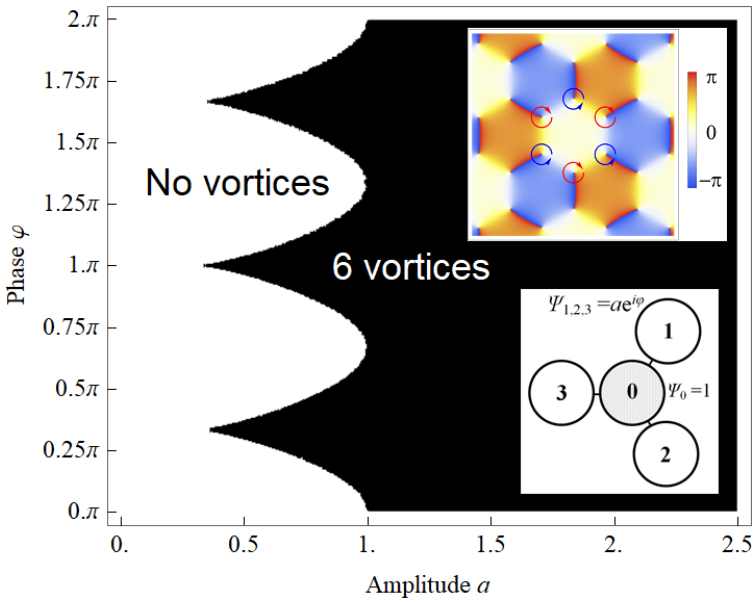

Figure S3: Formation of the regular 6 vortex pattern in momentum space (as in Fig. 3 of the main text) for the tetramer of C_{3v} symmetry. The central site has the wave function $\Psi_0 = 1$ and three neighbors have the wave function $\Psi_{1,2,3} = ae^{i\varphi}$ (see the lower inset). Black color in the phase diagram of parameters a and ϕ corresponds to the formation of vortices in momentum and white color is for their absence. If $a \gg 1$ then the Fourier image has a form described by $\tilde{\psi}(\mathbf{k}) = \sum_{\{j\}} e^{-i\mathbf{k}\mathbf{d}_j}$. This limit case describes the initial stages of C_{3v} symmetry WF evolution in the honeycomb lattice after excitation of a single site as discussed in the main text and simultaneously the hexagonal lattice with wave function located at three sites forming a triangle. The upper inset shows the phase pattern in momentum space for the case $a \gg 1$.